Worksheet 18, Math H53
Integration in Alternate Coordinate Systems

Thursday, April 4, 2013

1. Use polar coordinates to evaluate the integral $\iint_R \sin(x^2+y^2)\,dA$, where R is the region in the first quadrant between the circles with center the origin and radii 1 and 3.
2. Use polar coordinates to find the volume of the solid enclosed by the hyperboloid $-x^2-y^2+z^2=1$ and the plane $z=2$.
3. Evaluate the iterated integral $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$ by converting to polar coordinates.
4. A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north end. Find the volume of water in the pool.
5. Change from rectangular to cylindrical coordinates:
   (a) $(-1,1,1)$
   (b) $(-2,2\sqrt{3},3)$
   (c) $(2\sqrt{3},2,-1)$
   (d) $(4,-3,2)$
6. Sketch the solid described by the inequalities:
   (a) $0 \le r \le 2$, $-\pi/2 \le \theta \le \pi/2$, $0 \le z \le 1$.
   (b) $0 \le \theta \le \pi/2$, $r \le z \le 2$.
7. Use cylindrical coordinates to evaluate $\iiint_E (x+y+z)\,dV$, where E is the solid in the first octant that lies under the paraboloid $z = 4-x^2-y^2$.
8. Use cylindrical coordinates to find the volume of the solid that is enclosed by the cone $z=\sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=2$.
9. Change from rectangular to spherical coordinates:
   (a) $(0,-2,0)$
   (b) $(-1,1,-\sqrt{2})$
   (c) $(1,0,\sqrt{3})$
   (d) $(\sqrt{3},-1,2\sqrt{3})$
10. Sketch the solid described by the inequalities:
    (a) $2 \le \rho \le 4$, $0 \le \phi \le \pi/3$, $0 \le \theta \le \pi$.
    (b) $\rho \le 2$, $\rho \le \csc\phi$.
11. Use spherical coordinates to evaluate $\iiint_E (x^2+y^2)\,dV$, where E lies between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$.
12. Use spherical coordinates to evaluate $\iiint_E xyz\,dV$, where E lies between the spheres $\rho=2$ and $\rho=4$ and above the cone $\phi=\pi/3$.
13. Evaluate the integral

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx$$

by changing to spherical coordinates.