Worksheet 8, Math H53
Limits and Continuity in Functions of Several Variables

Thursday, February 21, 2013

1. Draw a contour map of the function $f(x, y) = ye^x$, showing several level curves.
2. Same as the previous problem, except with $g(x, y) = y/(x^2 + y^2)$.
3. Describe the level surfaces of the function $f(x, y, z) = y^2 + z^2$.
4. Find the limit, if it exists, or show that the limit does not exist:
   - $\lim_{(x,y)\to(1,-1)} e^{-xy} \cos(x + y)$
   - $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$
   - $\lim_{(x,y)\to(1,0)} \dfrac{xy - y}{(x - 1)^2 + y^2}$
   - $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y e^y}{x^4 + 4y^2}$
   - $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy + yz}{x^2 + y^2 + z^2}$
5. Use polar coordinates to show that $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2 + y^2)}{x^2 + y^2} = 1$.
6. Find $h(x, y) = g(f(x, y))$, where $g(t) = 1 + \ln t$ and $f(x, y) = (1 - xy)/(1 + x^2y^2)$. On which points (x, y) is h continuous?
7. Determine the set of points at which the function is continuous:
   - $F(x, y) = \dfrac{xy}{1 + e^{x-y}}$
   - $F(x, y, z) = \sin^{-1}(x^2 + y^2 + z^2)$
   - $F(x, y) = \begin{cases} \frac{x^2y^3}{2x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$
8. If $\mathbf{c} \in \mathbb{R}^n$, show that function f given by $f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$.