# Worksheet 7, Math H53

## Vector Functions

Tuesday, February 19, 2013

1. Sketch the curve of the vector function, and indicate with an arrow the direction in which $t$ increases.
   (a) $\mathbf{r}(t) = \langle \sin t, t \rangle$
   (b) $\mathbf{r}(t) = \langle t, t \rangle$
   (c) $\mathbf{r}(t) = \langle \cos t, \cos t \rangle$
   (d) $\mathbf{r}(t) = \langle t^3, t^2 \rangle$
2. Find a vector function that represents the curve of intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cyliner $y = x^2$.
3. Find the limit
$$\lim_{t \to 0} \left( \frac{t^2 - t}{t - 1}\mathbf{i} + \sqrt{t + 8}\mathbf{j} + \frac{\sin \pi t}{\ln t}\mathbf{k} \right).$$
4. Find the derivative of the vector function $\mathbf{r}(t) = \langle \tan t, \sec t, 1/t^2 \rangle$.
5. Find the unit tangent vector $\mathbf{T}(t)$ of the curve $\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle$ for $t = 1$, and in general.
6. Sketch the plane curve with vector equation $\mathbf{r}(t) = \langle 1 - \cos t, 2 + \sin t \rangle$, find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$, and sketch the tangent vector $\mathbf{r}'(\pi/6)$ and the vector $\mathbf{r}''(\pi/6)$ with tails starting at $\mathbf{r}(\pi/6)$. What physical interpretation do these vectors have?
7. Find the point on the curve $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, e^t \rangle$, $0 \leq t \leq \pi$ where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.
8. Find parametric equations for the tangent line to the curve $x = e^t$, $y = te^t$, $z = te^{t^2}$ at the point $(1, 0, 0)$.
9. If $\mathbf{r}$ is a vector function, find a simple expression for $\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)]$.
10. Find the length of the curve $\mathbf{r}(t) = \langle 1, t^2, t^3 \rangle$, $0 \leq t \leq 1$.
11. Reparametrize the curve $\mathbf{r}(t) = \langle e^{2t} \cos 2t, 2, e^{2t} \sin 2t \rangle$ with respect to arc length measured from the point where $t = 0$ in the direction of increasing $t$.