

Worksheet 3, Math H53

Equations of Lines and Planes

Tuesday, February 5, 2013

1. Find a vector equation and parametric equations for the line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t$, $y = 6 - 3t$, $z = 3 + 9t$.
2. Find parametric equations and symmetric equations for the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$.
3. Find parametric equations for the line through $(2, 4, 6)$ that is perpendicular to the plane $x - y + 3z = 7$. In what points does this line intersect the coordinate planes?
4. Find the equation of the plane through the point $(2, 4, 6)$ and parallel to the plane $z = x + y$.
5. Find the equation of the plane that passes through the points $(0, -2, 5)$ and $(-1, 3, 1)$, and is perpendicular to the plane $2z = 5x + 4y$.
6. Find the point at which the line $x = 3 - t$, $y = 2 + t$, $z = 5t$ intersects the plane $x - y + 2z = 9$.
7. Find the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$. You need not simplify once the answer is in the form of a number.
8. Find the point at which the lines described by $\mathbf{r}_1(t) = \langle 1, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$ and $\mathbf{r}_2(t) = \langle 2, 0, 2 \rangle + t \langle -1, 1, 0 \rangle$ intersect. Find an equation of the plane that contains these lines.
9. Find symmetric equations for the line of intersection of the planes given by $5x - 2y - 2z = 1$ and $4x + y + z = 6$.
10. Show that the distance between the parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ is given by

$$D = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}.$$

11. Give a geometric description of each family of planes, where the families are described by different values of the constant parameter.
 - (a) $x + y + z = c$, $c \in \mathbb{R}$
 - (b) $x + y + cz = 1$, $c \in \mathbb{R}$
 - (c) $y \cos \theta + z \sin \theta = 1$, $0 \leq \theta < 2\pi$