Worksheet 2, Math H53
Dot Products and Cross Products

Thursday, January 31, 2013

1. For vectors $\mathbf{a} = \langle 6, 0, -2 \rangle$ and $\mathbf{b} = \langle 0, 8, 0 \rangle$, find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

2. Do the same as in the previous problem, but for vector functions $\mathbf{a}(t) = t\mathbf{i} + \cos t\mathbf{j} + \sin t\mathbf{k}$ and $\mathbf{b}(t) = \mathbf{i} - \sin t\mathbf{j} + \cos t\mathbf{k}$.

3. Compute the vector $\mathbf{k} \times (\mathbf{i} - 2\mathbf{j})$ using properties of the cross product rather than determinants.

4. State whether each expression is meaningful. If not, explin why. If so, state whether it is a vector or a scalar.

   (a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

   (b) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$

   (c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

   (d) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

   (e) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

   (f) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

5. Determine whether the given vectors are orthogonal, parallel, or neither.

   (a) $\mathbf{a} = \langle -5, 3, 7 \rangle$, $\mathbf{b} = \langle 6, -8, 2 \rangle$

   (b) $\mathbf{a} = \langle 4, 6 \rangle$, $\mathbf{b} = \langle -3, 2 \rangle$

   (c) $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$

   (d) $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$

6. Suppose that $\mathbf{a} \neq \mathbf{0}$.

   (a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

   (b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

   (c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

7. Find the acute angles between the curves $y = \sin x$ and $y = \cos x$ in the domain $0 \le x \le \pi/2$ at their point of intersection. (The angle between two curves is defined as the angle between their tangent lines at the point of intersection.)

8. A tow truck drags a stalled car along a road. The chain makes an angle of $30°$ with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?

9. If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

10. Let points P, Q, R and S be defined by $P(-2, 1, 0)$, $Q(2, 3, 2)$, $R(1, 4, -1)$, $S(3, 6, 1)$. Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS.