This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. __T__   The r-combinations of a set S are counted by binomial coefficients.

b. __F__   A set with 6 elements has 40 subsets with more than two elements.

c. __F__   Pascal's Identity states that the sum of the binomial coefficients in the nth row of Pascal's triangle is equal to 2^n.

❀

Exercise. Prove the binomial identity, that for $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$$

We prove the identity by a combinatorial proof. Both sides of the identity count the number of subsets of an n-element set. The left hand side splits the count up by how many elements are in the subset; we know that $\binom{n}{k}$ counts the number of k-element subsets of an n-element set. Thus the left side sums over all sizes of subsets, and thus counts the number of subsets in total. On the other hand, we can count the subsets of an n-element set by independently choosing whether each of the n elements is included or excluded, giving n independent choices with 2 options each, or 2^n possibilities in total. This completes the proof.