Math 55 Quiz 8
October 19, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T A *double counting proof* is a type of combinatorial proof of an identity that uses counting arguments to prove that both sides of the identity count the same objects, but in different ways.

b. T For a nonempty finite set S, the number of subsets of S with an even number of elements is equal to the number of subsets of S with an odd number of elements.

c. F The coefficient of x in $(x + 2/x)^{21}$ is given by $2^{11}\binom{21}{10}$.

Exercise. We say that a poker hand of five cards is *prime* if it has exactly three cards which are number cards with a prime number value, that is, which are one of 2, 3, 5 or 7 of any suite. How many different prime poker hands are possible?

There are 16 cards which are prime numbered of any suite, so to choose a prime hand, we need to pick any three cards from these sixteen for the prime cards, and then any two cards from the remaining 36 non-prime cards. In total this gives us $\binom{16}{3} \cdot \binom{36}{2}$ prime poker hands.