This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. __T__ If A and B are sets such that $|A| = m$ and $|B| = n$, then there are n^m different functions $f : A \to B$.

b. __F__ The empty tree is one example of a full binary tree.

c. __T__ Define a set S by the following recursive definition. Base step: $2 \in S$ and $1/2 \in S$. Recursive step: If x and y are in S, then $xy \in S$. Then the set S consists of all powers of 2.

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Exercise. Suppose that p and q are distinct prime numbers, and that $n = pq$. Find the number of positive integers not exceeding n that are relatively prime to n.

A number is relatively prime to n exactly if it is divisible by neither of p and q. We can count this collection of numbers by counting the numbers that are divisible by p or by q, and subtracting this amount from n, the total number of numbers not exceeding n. There are q numbers divisible by p, and p numbers divisible by q, and one number (just n) divisible by both. Thus the count is $p+q-1$ for numbers divisible by p or q, and the number of relatively prime numbers is $n-p-q+1$.