Math 55 Quiz 4
September 21, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T If f is a function from A to B, and S and T are subsets of B, then

$$f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T).$$

b. T There are six integers n between 0 and 50 such that $n \equiv 4 \pmod 9$.

c. F The Cantor diagonalization argument shows that the cardinality of $\mathbb{N}$ is equal to the cardinality of $\mathbb{Q}$.

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Exercise. Find a solution to the recurrence

$$a_n = 2a_{n-1} + 1, \quad a_0 = 0,$$

and show that your formula for a_n satisfies the recurrence.

Finding the first few values of a_n from the recurrence, we have

$a_0 = 0$, $a_1 = 2 \cdot a_0 + 1 = 1$, $a_2 = 2 \cdot a_1 + 1 = 3$, $a_3 = 2 \cdot a_2 + 1 = 7$,

$a_4 = 2 \cdot a_3 + 1 = 15$, $a_5 = 2 \cdot a_4 + 1 = 31$, ...

The general pattern appears to be $a_n = 2^n - 1$. To check that this satisfies the recurrence, we note that

$$2^0 - 1 = 0 = a_0,$$

and

$$2 \cdot (2^{n-1} - 1) + 1 = 2^n - 2 + 1 = 2^n - 1.$$

Thus the formula $2^n - 1$ satisfies the recurrence.