Math 55 Quiz 4
September 21, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. _T_ The sequence $a_n = n^2$ is a solution to the recurrence $a_{n+1} = a_n + 2n + 1$.

b. _T_ There exists a modulus m and an integer $d \not\equiv 0 \pmod{m}$ such that d does *not* have a multiplicative inverse in modular arithmetic mod m.

c. _T_ The function $f : \mathbb{N} \to \mathbb{Z}$ given by $f(n) = n^2 - 3$ is injective.

✳

Exercise. Give an example of two uncountable sets A and B such that $A \cap B$ is countably infinite.

(Remember: "Give an example" means to define some sets A and B, and to prove that they satisfy the desired properties.)

Let $A = (0,1) \cup \mathbb{N}$, and let $B = (1,2) \cup \mathbb{N}$. Then A is uncountable since $(0,1) \subseteq A$ and $(0,1)$ is uncountable, and B is uncountable similarly with the interval $(1,2)$. However $A \cap B = \mathbb{N}$, which is countably infinite.