Math 54 Quiz 7
March 20th, 2014

1. Show that $\mathbf{a}_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$, and $\mathbf{a}_3 = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}$ form an orthogonal set.

   Show that $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, and $\mathbf{b}_3 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$ form an orthogonal set.

   In $\mathbb{R}^3$, find the matrix of the transformation that reflects a vector $\mathbf{x}$ across the plane spanned by $\mathbf{b}_1, \mathbf{b}_2$, in the basis $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$

2. Show that the product of two orthogonal matrices is orthogonal.

3. True or False.

   (a) Every square matrix has a complex eigenvalue.
   (b) If $\mathbf{x}$ and $\mathbf{y}$ are vectors and $\mathbf{x} \cdot \mathbf{y} = 0$, then $\mathbf{x}$ and $\mathbf{y}$ are linearly independent.

   (c) The product of two diagonalizable matrices is diagonalizable

   (d) Every 2×2 matrix with two distinct eigenvalues is diagonalizable