Math 54 Quiz 5
Feb. 27, 2014

This quiz will be graded out of 20 points. Read each problem carefully and show your work.

1. (10 points) Let $\mathbb{P}_2$ be the space of degree 2 polynomials. Define the map $T : \mathbb{P}_2 \to \mathbb{R}^2$ by
$$T(p) = \begin{bmatrix} p(-1) \\ p(1) \end{bmatrix}.$$

   (a) Find a nonzero element of the kernel of T.

   (b) Find a nonzero element of the range of T.

   Solution:
   In order for an element to be in the kernel, $p(-1) = p(1) = 0$. So we are looking for a polynomial that has roots as ± 1. For example, $(x-1)(x+1)$ is in the nullspace.

   An example of an element of the range would be $T(x+1) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

2. (10 points) Find a basis for the nullspace and column space of this matrix

$$A = \begin{bmatrix} 1 & 1 & -2 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -8 & 0 & 16 \end{bmatrix}$$

Solution:
The first three columns are linearly independent, so they form a basis for the column space.
Solving the equation $Ax = 0$ tells us that the elements

$$\left\{ \begin{bmatrix} -3 \\ 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

are both in the nullspace. They are linearly independent as the 4th entry is 0 in the first element, but nonzero in the second. Finally, they span the nullspace by the rank-nullity theorem.