Math 53 Quiz 7
March 22, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). No calculators, please! Please read the instructions carefully, and explain your work.

Problem 1. (3 points) Compute the douple integral

$$\iint_D xye^{y^2}\, dA$$

over the domain $D = [0,2] \times [0,1]$.

This double integral can be computed as an iterated integral:

$$\int_0^1 \int_0^2 x y e^{y^2} dx\, dy = \int_0^1 y e^{y^2} \int_0^2 x\, dx\, dy$$

$$= \int_0^1 2y e^{y^2} dy = e^{y^2}\Big|_0^1 = e - 1.$$

Problem 2. (3 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = 3x + y$ subject to the constraint $x^2 + y^2 = 10$.

Rewriting the constraint as $g(x,y) = x^2 + y^2 = 10$, we want to find solutions to the system of equations $\nabla f = \lambda \nabla g$ and $g(x,y) = 10$. We compute $\nabla f = \langle 3, 1 \rangle$ and $\nabla g = \langle 2x, 2y \rangle$, so we get

$$\begin{cases} 3 = 2\lambda x \\ 1 = 2\lambda y \\ x^2 + y^2 = 10 \end{cases}$$

Clearly we can't have $\lambda = 0$ in this system, so we rewrite $x = \frac{3}{2} \cdot \frac{1}{\lambda}$, $y = \frac{1}{2} \cdot \frac{1}{\lambda}$, and find $x^2 + y^2 = \left(\frac{9}{4} + \frac{1}{4}\right) \cdot \frac{1}{\lambda^2} = 10$, so $\lambda = \pm 1/2$. From this and the first two equations, we get the solutions $(x,y) = (3,1)$ and $(x,y) = (-3,-1)$. Plugging in to f, we have that $f(3,1) = 10$ is the maximum, and $f(-3,-1) = -10$ is the minimum.

Problem 3. (4 points) Find the absolute maximum of

$$f(x, y) = y \sin x$$

on the domain $D = \{(x, y) : 0 \le x \le \pi/2, 0 \le y \le \cos x\}$.

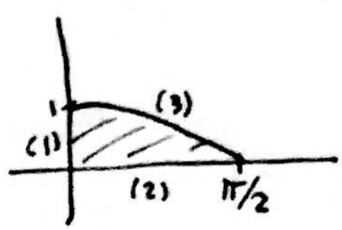
We find the critical points of f, optimize on the boundary, and then compare all of the values obtained to find the maximum.

The critical points of f are the points satisfying

$$\nabla f = \langle y \cos x, \sin x \rangle = \langle 0, 0 \rangle.$$

The solutions to this system are given by the points $(x,y) = (k\pi, 0)$ for k an integer, and in particular the only such point in D ~~are~~ is $(0,0)$.

The domain D can be represented by the sketch



Along edges (1) and (2), f takes value 0 at every point, so we just need to optimize along edge (3). Along this edge, y is a function of x, so we can optimize $g(x) = f(x, \cos(x)) = \cos(x)\sin(x) = \frac{1}{2}\sin(2x)$. Then $g'(x) = \cos(2x)$, so $g'(x) = 0$ when $x = \pi/4$. Thus the critical point of this function ~~has~~ value $g(\pi/4) = \frac{1}{2}$, and boundary values $g(0) = g(\pi/2) = 0$. We see that f is maximized at $(x,y) = (\pi/4, \frac{\sqrt{2}}{2})$ with max value $1/2$.