Math 53 Quiz 5

February 22, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). No calculators, please! Please read the instructions carefully, and explain your work.

Problem 1. (3 points) If $f(x, y) = \ln(2x + 3y)$, compute the second partial derivative $f_{xy}(3, 2)$.

The first partial derivative $f_x(x,y)$ is given by

$$f_x(x,y) = \frac{\partial}{\partial x}\ln(2x+3y) = \frac{1}{2x+3y}\frac{\partial}{\partial x}(2x+3y) = \frac{2}{2x+3y}$$

The second partial derivative $f_{xy}(x,y)$ is then

$$f_{xy}(x,y) = \frac{\partial}{\partial y} f_x(x,y) = \frac{\partial}{\partial y}\frac{2}{2x+3y} = \frac{-2}{(2x+3y)^2}\frac{\partial}{\partial y}(2x+3y) = \frac{-6}{(2x+3y)^2}$$

Then $f_{xy}(3,2) = {-6}/{(2\cdot 3+3\cdot 2)^2} = -1/24$.

Problem 2. (3 points) Prove that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x\sin(y)}{x^2+\sin^2(y)}$$

Taking the limit along the path $y=0$ gives a limiting value of

$$\lim_{x\to 0}\frac{x\sin(0)}{x^2+\sin^2(0)} = \lim_{x\to 0}\frac{0}{x^2} = 0$$

while taking the limit along the path $x=\sin(y)$ gives

$$\lim_{y\to 0}\frac{(\sin(y))\sin(y)}{(\sin(y))^2+\sin^2(y)} = \lim_{y\to 0}\frac{\sin^2(y)}{2\sin^2(y)} = \lim_{y\to 0}\frac{1}{2} = \frac{1}{2}.$$

Since the limits along different paths approaching $(0,0)$ are not the same, the limit doesn't exist.

Problem 3. (4 points) Prove that the function

$$f(x, y) = \begin{cases} \frac{(x+y)^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0) \end{cases}$$

is not continuous at the origin.

If f is continuous at $(0,0)$, then we must have that

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 1.$$

In particular, this would imply that the limit of f along any path approaching the origin would also be 1. However, if we consider the path $y=x$, then the limit along this path is

$$\lim_{x\to 0} \frac{(x+(x))^2}{x^2+(x)^2} = \lim_{x\to 0} \frac{4x^2}{2x^2} = \lim_{x\to 0} 2 = 2.$$

This limit is not equal to $f(0,0)=1$, so we conclude that f is not continuous at the origin.