Math 53 Quiz 5.5 (Just for Practice)
March 1, 2017

This quiz is a practice quiz, and will not be graded. For the most authentic experience, set aside 25 minutes to work, and put away all outside resources, as well as devices such as calculators. Please read the instructions carefully, and explain your work.

Problem 1. (0 points) Find the linear approximation of the function $f(x, y) = \ln(1 - (x^2 + y^2))$ at the point $P = (0, 0)$.

To find the linear approximation, we start by computing the partial derivatives f_x and f_y:

$$f_x(x,y) = \frac{1}{1-(x^2+y^2)} \cdot (-2x), \qquad f_y(x,y) = \frac{1}{1-(x^2+y^2)} \cdot (-2y)$$

Then the linear approximation is given by

$$L_{(0,0)}(x,y) = f_x(0,0)(x-0) + f_y(0,0)(y-0) + f(0,0)$$
$$= \frac{-2(0)}{1-(0^2+0^2)} x + \frac{-2(0)}{1-(0^2+0^2)} y + \ln(1-(0^2+0^2)) = 0.$$

Problem 2. (0 points) The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.5 L and the temperature decreases from 310 K to 300 K.

If we rewrite the relation as $P = 8.31 \cdot \frac{T}{V}$, we compute the differential as

$$dP = 8.31 \cdot \frac{1}{V} \cdot dT - 8.31 \cdot \frac{T}{V^2} \cdot dV$$

Using the values $V = 12$, $dV = 0.5$, $T = 310$, $dT = -10$, we get an approximate change of

$$dP = 8.31 \cdot \left(\frac{-10}{12} - \frac{310 \cdot 0.5}{12^2} \right) = 8.31 \cdot \left(\frac{-120 - 155}{144} \right)$$

$$= -8.31 \cdot \frac{275}{144}.$$

Problem 3. (0 points) Find the second derivative of y with respect to x for the curve described by $e^{x+y} = x - y$.

Considering y as a function of x, we use implicit differentiation to compute that

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(x-y)$$

$$\rightarrow e^{x+y}(1+y') = 1-y'$$

$$\rightarrow (e^{x+y}+1)y' = 1-e^{x+y}$$

$$\rightarrow y' = \frac{1-e^{x+y}}{1+e^{x+y}}.$$

A second implicit differentiation gives us that

$$y'' = \frac{(1+e^{x+y})\cdot(-e^{x+y}(1+y')) - (1-e^{x+y})(e^{x+y}(1+y'))}{(1+e^{x+y})^2}$$

$$= \frac{-2e^{x+y}(1+y')}{(1+e^{x+y})^2} = \frac{-2e^{x+y}(2/(1+e^{x+y}))}{(1+e^{x+y})^2}$$

$$= \frac{-4e^{x+y}}{(1+e^{x+y})^3}.$$