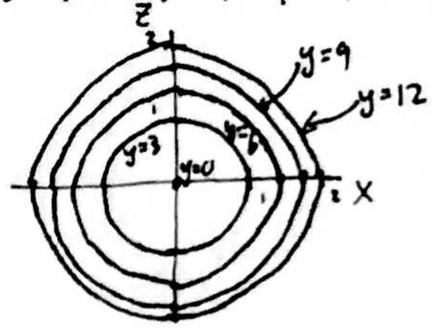
# Math 53 Quiz 4

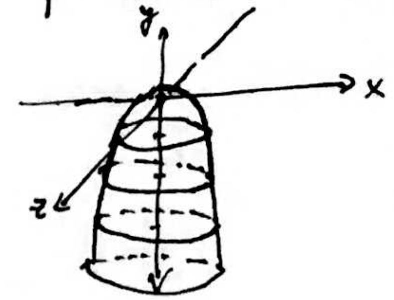
February 15, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). Please read the instructions carefully, and explain your work. No calculators, please!

**Problem 1.** (3 points) Sketch several traces of $3x^2 + y + 3z^2 = 0$ in $y$, and use these to sketch the surface. What type of surface is it?

Traces in $y$ are the $x$-$z$ graphs of the equation obtained by setting $y$ to a particular constant value. We rewrite the equation as $x^2+z^2=-y/3$, and see that the traces are circles of radius $\sqrt{-y/3}$ for $y \geq 0$. This means that the surface is a paraboloid in the neg. $y$ direction.



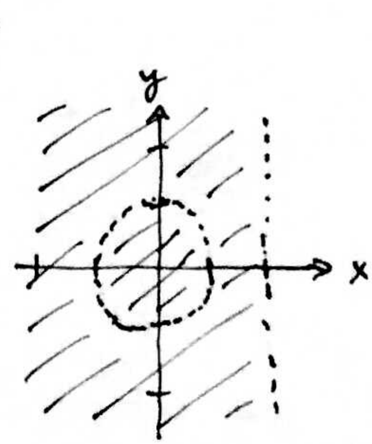

**Problem 2.** (3 points) Find and sketch the (maximal) domain of the function

$$g(x, y) = \frac{\ln(2 - x)}{1 - x^2 - y^2}$$

$g$ is properly defined when the expression inside the natural logarithm is positive and the denominator in the fraction is nonzero. That is, we require that $2-x>0 \rightarrow x<2$ and $1-x^2-y^2 \neq 0 \rightarrow x^2+y^2 \neq 1$. This is the domain

$$D = \{(x,y) \in \mathbb{R}^2 : x<2 \text{ and } x^2+y^2 \neq 1\}$$

which consists of all points to the left of the line $x=2$, excepting the unit circle centered at the origin. Sketched, the domain looks like this:



**Problem 3.** (4 points) Find a vector function that represents the curve of intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

If we use the parameter $t$, then let $x$ just take value $t$: $x(t) = t$. To satisfy the equation of the parabolic cylinder, we need for $y(t) = x(t)^2 = t^2$, and to satisfy the equation of the paraboloid, we need to set $z(t) = 4x(t)^2 + y(t)^2 = 4t^2 + t^4$. Thus the overall vector function for this curve is just

$$\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle.$$