Math 53 Quiz 1
January 25, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). Please read the instructions carefully, and explain your work.

Problem 1. (3 points) Eliminate the parameter to find a Cartesian equation for the curve, and sketch the curve, indicating with an arrow the direction in which the curve is traced as the parameter increases.

$$x = e^t, \quad y = \cosh t$$

(Recall that the hyperbolic cosine function cosh is given by $\cosh t = (e^t + e^{-t})/2$.)

We can write y in terms of x by using that $e^{-t} = 1/e^t = 1/x$.
so

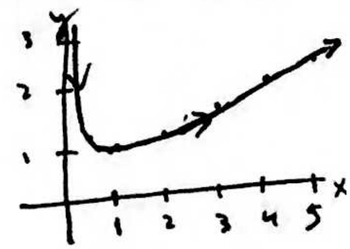
$$y = \frac{1}{2}(e^t + e^{-t}) = \frac{1}{2}\left(x + \frac{1}{x}\right) = \frac{x^2+1}{2x}.$$

Since $x = e^t$ only takes positive values, the curve is given by solutions of the above equation for $x > 0$. A few values are:

x	y
1/2	5/4
1	1
2	5/4

x	y
3	5/3
4	17/8
5	26/10

Sketched, this looks like:



Problem 2. (3 points) Find an equation of the tangent line to the curve

$$x = \sin \pi t, \quad y = t^2 + t$$

at the point $(0, 2)$.

First we should find the value of t corresponding to the point $(0,2)$. Since $x = \sin \pi t = 0$, t must be an integer, and since $y = t^2 + t = 2$, we see that t may be either 1 or -2.
We will find an equation for the curve at $t=1$. At this point, the slope is given by

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{2t+1}{\pi \cos(\pi t)}, \text{ and at } t=1, \text{ this is } \frac{3}{-\pi} = -\frac{3}{\pi}.$$

Thus the equation of this tangent line is: $y - 2 = -3/\pi \cdot x$.

Problem 3. (4 points) Set up (but don't evaluate) an integral that represents the area in the first quadrant cut out by the curve

$$x = t^3 - 3t^2 + 2t, \quad y = e^3 - e^t$$

The curve is in the first quadrant when both x and y are nonnegative. y is nonnegative for $t \leq 3$, and $x = t \cdot (t-1) \cdot (t-2)$ is nonnegative for $0 \leq t \leq 1$ and $t \geq 2$. Thus the curve is in the first quadrant for two stretches, for $0 \leq t \leq 1$ and $2 \leq t \leq 3$.

Since y is decreasing in t, we can think of x as a function of y and write the area as the area between the curve and the y-axis:

$$A = \int_1^0 x(t)\, y'(t)\, dt + \int_3^2 x(t)\, y'(t)\, dt$$

$$= \int_1^0 (t^3 - 3t^2 + 2t) \cdot (-e^t)\, dt + \int_3^2 (t^3 - 3t^2 + 2t) \cdot (-e^t)\, dt .$$