Worksheet 10, Math 53
Change of Variables

Wednesday, October 31, 2012

1. Find the image of the triangular region with vertices $(0,0)$, $(2,2)$, and $(0,2)$ under the transformation $x = u^2$, $y = v$.

2. Let E be the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$
Use the change of variables $x = au$, $y = bv$ and $z = cw$ to compute the volume of E.

3. Let Q be the quadrilateral in the xy-plane with vertices $(1,0)$, $(4,0)$, $(0,1)$, and $(0,4)$. Evaluate
$$\iint_Q \frac{1}{x+y} dA$$
with the change of variables $x = u - uv$ and $y = uv$.

4. Use a change of variables to evaluate the integral
$$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA,$$
where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,2)$, and $(0,1)$.

5. Find equations for a transformation from a rectangular region in the uv-plane into the parallelogram in the xy-plane with vertices $(0,1)$, $(4,3)$, $(2,4)$, and $(-2,1)$.

6. Find equations for a transformation from a rectangular region in the uv-plane into the region in the xy-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.