Worksheet 7, Math 53
Gradients and Optimization

Wednesday, October 10, 2012

1. Find the equation of the tangent plane and the normal line to the surface
$$x + y + z = e^{xyz}$$
at the point $(0, 0, 1)$.

2. Find the equation of the tangent plane to the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
at the point (x_0, y_0, z_0).

3. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r, using both the method of derivative tests and the method of Lagrange multipliers.

4. Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin, using both the method of derivative tests and the method of Lagrange multipliers.

5. Find the extreme values of $f(x, y) = x^2 + y^2 + 4x - 4y$ on the region $x^2 + y^2 \leq 9$.

6. Find the maximum value of
$$f(x_1, x_2, \ldots, x_n) = \sqrt[n]{x_1 x_2 \cdots x_n}$$
given that $x_1, x_2, \ldots, x_n$ are positive numbers and $x_1 + x_2 + \cdots + x_n = c$, where c is a constant. Deduce that for positive $x_1, x_2, \ldots, x_n$ we have
$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}.$$
This means the geometric mean of n numbers is no larger than the arithmetic mean of these numbers. When are these two means equal?