Worksheet 5, Math 53

Limits and Derivatives in Multiple Dimensions

Wednesday, September 26, 2012

1. Determine the set of points at which the function is continuous:

   (a) $f(x,y) = \dfrac{1+x^2+y^2}{1-x^2-y^2}$

   (b) $f(x,y) = \tan^{-1}\left(\left(\dfrac{1}{x+y}\right)^2\right)$

   (c) $f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$

2. If $\mathbf{c}$ is a fixed vector in $\mathbb{R}^n$, show that the function f given by $f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$.

3. If $f(x,y,z) = \sqrt{1+xz} + \sqrt{1-xy}$, find f_{xyz}. Can you think of a way to do this computation in your head?

4. A friend tells you that there is a function f whose partial derivatives are $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$. Is your friend a dirty liar?

5. If a, b, c are the sides of a triangle, and A, B, C are the angles opposite the respective sides, find $\partial A/\partial a$, $\partial A/\partial b$, and $\partial A/\partial c$ by implicit differentiation of the law of cosines.

6. How many nth-order partial derivatives does a function of two variables have? If these partial derivatives are all continuous, how many of them can be distinct?