# Worksheet 1, Math 53
## Parametric Equations

Wednesday, August 29, 2012

1. For the following parametric equations, sketch the curve represented by the equations, indicating with an arrow the direction in which the curve is traced as the parameter increases, and eliminate the parameter to find a Cartesian equation of the curve.

   (a) $x = 1 - t^2$, $y = t - 2$, $-2 \leq t \leq 2$

   (b) $x = \sin t$, $y = \csc t$, $0 < t < \pi/2$

   (c) $x = e^{2t}$, $y = t + 1$

   (d) $x = \tan^2 \theta$, $y = \sec \theta$, $-\pi/2 < \theta < \pi/2$

2. Suppose a curve is given by the parametric equations $x = f(t)$, $y = g(t)$, where the range of $f$ is $[0, 4]$ and the range of $g$ is $[-2, 1]$. What can you say about the curve?

3. Compare the curves represented by the following parametric equations. How do they differ?

   (a) $x = t^3$, $y = t^2$

   (b) $x = t^6$, $y = t^4$

   (c) $x = e^{-3t}$, $y = e^{-2t}$

4. Let $P$ be a point at a distance $d$ from the center of a circle of radius $r$, and consider the *trochoid* traced out by $P$ as the circle rolls along a straight line. Assuming that

   - The circle starts out sitting on the origin,
   - The circle rolls in the negative direction along the $x$-axis,
   - The circle rolls at a rate of $\theta$ radians per unit time, and
   - The point $P$ starts out directly above the center of the circle,

   find the parametric equations of $P$ in terms of the time parameter $t$.