Math 480A2, Homework 4
Due September 22, 2022

Homework is graded out of a total of 10 points. Collaboration is permitted, but you must list all coauthors on a problem's solution at the top of the page, and your writing must be your own.

For the following problems, let $F_2 = \mathbb{Z}/2\mathbb{Z}$ denote the binary field.

Problem 1. (3 points) Find the factorization of $x^{16} - x$ into irreducible factors over F_2. (*Hint:* You may make use of solutions from previous homework assignments.)

Problem 2. (2 points) Describe a finite field K of order 16 over F_2, presented as the quotient of $F_2[x]$ by an irreducible polynomial. Describe the general form of an element of K as a linear combination of small powers of $\bar{x}$ in the quotient, and give the relation satisfied by $\bar{x}^4$.

Problem 3. (3 points) Find a multiplicative generator of $K^\times$ by identifying a nonzero element $\alpha \in K$ satisfying $\alpha^3 \neq 1$ and $\alpha^5 \neq 1$.

Problem 4. (2 points) Let $\beta = \alpha^5$, where α is the multiplicative generator found in the last problem. Show that the set $K' = \{0, 1, \beta, \beta^2\}$ is a subfield of K of order 4 by writing out the addition and multiplication tables of these elements.

Challenge. (1 bonus point) Describe an explicit isomorphism between K' and $F_2[y]/(y^2 + y + 1)$.