Math 480A2, Homework 3
Due September 15, 2022

Homework is graded out of a total of 10 points. Collaboration is permitted, but you must list all coauthors on a problem's solution at the top of the page, and your writing must be your own.

Problem 1. (3 points) Compute the monic irreducible polynomials of degree 4 over the binary field $F_2 = \mathbb{Z}/2\mathbb{Z}$. (*Hint:* if a polynomial of degree 4 is reducible, it either has a linear factor and thus a root in F_2, or it can be written as a product of monic irreducible polynomials of degree 2.)

Problem 2. (2 points) Let $F_3 = \mathbb{Z}/3\mathbb{Z}$, and let $f(x) = x^4 + x - 1 \in F_3[x]$. Then f is irreducible over F_3, so the quotient $K = F_3[x]/(x^4 + x - 1)$ is a field. Compute the inverse of $x^2 + 1$ in K.

Challenge. (1 bonus point) Prove that $x^4 + x - 1$ is irreducible over F_3.

Problem 3. (3 points) Prove that the following numbers are algebraic over $\mathbb{Q}$ by finding a nonzero polynomial with rational coefficients for which they are a root: $1 + \sqrt{2}$, $\sqrt{1 + \sqrt{2}}$, $\sqrt{2} + \sqrt{3}$.

Problem 4. (2 points) Compute (with proof) the irreducible polynomial of the golden ratio $\phi = (1 + \sqrt{5})/2$ over $\mathbb{Q}$.