Worksheet 8, Math 1B
Series Representations of Functions

Friday, March 9, 2012

1. (Reprinted from WS 7) Find the sums of the following series using differentiation, integration, and summation:

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^{n-1}, \quad |x| < 1$

   (b) $\displaystyle\sum_{n=1}^{\infty} nx^{n}, \quad |x| < 1$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

   (d) $\displaystyle\sum_{n=2}^{\infty} n(n-1)x^{n}, \quad |x| < 1$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{2^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

2. Uses series to evaluate the limit

$$\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$$

3. Show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   is not equal to its Maclaurin series.

4. Find the sums of the following series by comparing to known series:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

   (c) $1 - \ln x + \dfrac{(\ln x)^2}{2!} - \dfrac{(\ln x)^3}{3!} + \cdots$