Worksheet 7, Math 1B
Power Series

Monday, March 5, 2012

1. Find the radius of convergence and interval of convergence of the series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4^n}(x+1)^n$

2. Suppose that the radius of convergence of the power series $\sum c_n x^n$ is R. What is the radius of convergence of the power series $\sum c_n x^{2n}$?

3. Find a power series representation for the function and determine the radius of convergence:

   (a) $f(x) = \dfrac{1+x}{1-x}$

   (b) $f(x) = \ln(5 - x)$

   (c) $f(x) = \dfrac{x^3}{(x-2)^2}$

4. Find the sums of the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^{n-1}, \quad |x| < 1$

   (b) $\displaystyle\sum_{n=1}^{\infty} nx^n, \quad |x| < 1$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

   (d) $\displaystyle\sum_{n=2}^{\infty} n(n-1)x^n, \quad |x| < 1$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{2^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

5. Fix an integer $k > 0$, and let $f(x) = \sum_{n=0}^{\infty} c_n x^n$, where $c_{n+k} = c_n$ for all $n \geq 0$. Assume that f is not a constant function. Find the interval of convergence of the series, and a formula for $f(x)$.