Worksheet 4, Math 1B

Numerical Integration, Improper Integrals

Friday, February 3, 2012

1. Consider the integral $\int_1^2 e^{1/x}\,dx$. How large do we have to choose n so that, respectively, the trapezoid and midpoint approximations with n subintervals are accurate within an error bound of 10^{-4}?

2. Sketch the graph of a continuous function on $[0, 2]$ for which the right endpoint approximation with $n = 2$ is more accurate than Simpson's Rule.

3. Find the *escape velocity* v_0 that is needed to propel a rocket of mass m out of the gravitational field of a planet with mass M and radius R. Use Newton's Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2},$$

and the fact that the initial kinetic energy of $\frac{1}{2}mv_0^2$ supplies the needed work.

4. Show that if $a > -1$ and $b > a + 1$, then the following integral is convergent:

$$\int_0^\infty \frac{x^a}{1 + x^b}\,dx.$$

5. Show that if f is a polynomial of degree 3 or lower, then Simpson's Rule gives the exact value of $\int_a^b f(x)\,dx$.

6. Find the value of the constant C for which the integral

$$\int_0^\infty \left(\frac{x}{x^2 + 1} - \frac{C}{3x + 1}\right)\,dx$$

converges. Evaluate the integral for this value of C.