

Worksheet 11, Math 1B

More Differential Equations

Monday, April 9, 2012

1. Classify the following differential equations according to order, homogeneity, linearity, and separability:

(a) $y' = 3xy$

(b) $y' = 3y^2$

(c) $y'' = 3y + x$

(d) $y''' + 3y' + xy = 2x^2$

(e) $y'''y'' = 1$

(f) $y' - y = 0$

(g) $x^2 + 2x \cos(x) = y'$

(h) $e^x y'' = e^{y''} x$

(i) $x \ln(x)y' + y = xe^x$

(j) $y' - y = (x + 1)(x - 1)y$

2. Find the general solution of the differential equation, then find a specific solution with the given conditions:

(a) $y' = x + y$, $y(0) = 2$.

(b) $ty' + 2y = t^3$, $t > 0$, $y(1) = 0$.

(c) $xy' = y + x^2 \sin x$, $y(\pi) = 0$.

(d) $4y'' - 4y' + y = 0$, $y(0) = 1$, $y'(0) = -1.5$.

(e) $y'' - 2y' + 5y = 0$, $y(\pi) = 0$, $y'(\pi) = 2$.

(f) $y'' + 2y' = 0$, $y(0) = 1$, $y(1) = 2$.

3. Let L be a nonzero real number, and consider the boundary value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$. For the cases of $\lambda = 0$ and $\lambda < 0$, show that the problem has only the trivial solution $y = 0$. For the case of $\lambda > 0$, find the values of λ for which this problem has a nontrivial solution and give the corresponding solution.