Worksheet 1, Math 1B
Integration by Parts, and Trigonometric Integrals

Friday, January 20, 2012

1. Evaluate the following integrals:
   (a) $\int \cos x \, \ln(\sin x)\, dx$
   (b) $\int \sin(\ln x)\, dx$
   (c) $\int_{\pi/6}^{\pi/3} \csc^3 x \, dx$

2. If $f(0) = g(0) = 0$ and f'' and g'' are continuous, show that
$$\int_0^a f(x)g''(x)\, dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\, dx.$$

3. If f and g are inverse functions and f' is continuous, prove that
$$\int_a^b f(x)\, dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\, dy.$$

4. Find the volume obtained by rotating the region bounded by the curves
$$y = \sin^2 x, \quad y = 0, \quad 0 \le x \le \pi$$
about the x-axis.

5. Prove that for positive integers m and n,
$$\int_{-\pi}^{\pi} \sin mx \, \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}.$$

6. A *finite Fourier series* is given by the sum
$$f(x) = \sum_{n=1}^{N} a_n \sin nx = a_1 \sin x + a_2 \sin 2x + \cdots + a_N \sin Nx,$$
where the coefficients a_i for $i = 1, 2, \ldots, N$ are given numbers. Show that the mth coefficient a_m is given by the formula
$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx \, dx.$$