Quiz 3 Solutions, Math 1B, Section 310

Friday, March 2, 2012

Name:

Student ID#:

Please place personal items under your seat.
No use of notes, texts, calculators, or fellow students is allowed.
Show all of your work in order to receive full credit.

Please pick your favorite **four** out of the following **five** problems to solve. Circle the numbers for the problems that you choose.

Do the following series converge or diverge? Justify your answer.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{(n+1)10^n}$

   Solution

   Since there is a simple exponential term mixed in with polynomial terms, we apply the ratio test.

$$\lim_{n\to\infty} \left| \left(\frac{3(n+1)^2}{(n+2)10^{n+1}} \right) \Big/ \left(\frac{3n^2}{(n+1)10^n} \right) \right| = \lim_{n\to\infty} \frac{1}{10} \cdot \left(\frac{n+1}{n} \right)^2 \cdot \left(\frac{n+1}{n+2} \right) = \frac{1}{10}$$

   Since the limit is less than 1, we conclude that the series converges.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{(2n+3)^3}$

   Solution

   Since the terms of the series are a ratio of polynomials, with only positive terms, we try the limit comparison test with $1/n^2$.

$$\lim_{n\to\infty} \frac{(n+1)/(2n+3)^3}{1/n^2} = \lim_{n\to\infty} \frac{n^3+n^2}{8n^3+36n^2+54n+27} = \frac{1}{8}$$

   Since the limit is positive and finite, we conclude that the series converges since $\sum_{n=1}^{\infty} 1/n^2$ converges as a p-series with $p > 1$.

3. $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

Solution

Since the sum involves a function of n raised to a power which is a function of n, we try the root test.

$$\lim_{n\to\infty}\left|\left(1+\frac{1}{n}\right)^{n^2}\right|^{1/n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e,$$

where the final equality is an identity covered in class. Thus since the limit is greater than 1, we conclude that the series diverges.

4. $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{\sqrt{n}}$

Solution

Since $n+1 > \sqrt{n}$ for each $n \geq 1$, we see that each term in the series is at least 1. Thus, since the terms of the series do not converge to 0, the Test for Divergence gives us that the series diverges.

5. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sqrt{n}}{n+2}$

Solution

The terms of this series alternate, so we attempt to apply the alternating series test. Letting $f(x) = \sqrt{x}/(x+2)$, we have

$$f'(x) = \frac{(x+2)/(2\sqrt{x}) - \sqrt{x}}{(x+2)^2} = \frac{1/\sqrt{x} - \sqrt{x}/2}{(x+2)^2},$$

and this is negative for $x > 2$. Since f is eventually decreasing, the sequence of terms $a_n = f(n)$ are as well. We also have

$$lim_{n\to\infty}\frac{\sqrt{n}}{n+2} \stackrel{H}{=} \lim_{n\to\infty}\frac{1}{2\sqrt{n}} = 0.$$

Thus the series satisfies the necessary conditions for the alternating series test, and we conclude that the series converges.