

Worksheet 5, Math 1A

Mean Value Theorem, Graph Sketching

Friday, November 1, 2013

1. Verify that $f(x) = x^3 - 3x + 2$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-2, 2]$, and find all numbers c that satisfy the conclusion of the Mean Value Theorem.
2. Show that the equation $2x + \cos x = 0$ has exactly 1 real root.
3. (Warning: Tricky) Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.
4. Suppose that $3 \leq f'(x) \leq 5$ for all values of x . Show that $18 \leq f(8) - f(2) \leq 30$.
5. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x ? If so, give an example. If not, prove it.
6. Show that $\sqrt{1+x} < 1 + x/2$ if $x > 0$.
7. Sketch the graph of a function f with the following properties:
 - (a) $f'(x) > 0$ if $|x| < 2$
 - (b) $f'(x) < 0$ if $|x| > 2$
 - (c) $f'(2) = 0$
 - (d) $\lim_{x \rightarrow \infty} f(x) = 1$
 - (e) $f(-x) = -f(x)$
 - (f) $f''(x) < 0$ if $0 < x < 3$
 - (g) $f''(x) > 0$ if $x > 3$
8. Let $f(x) = 200 + 8x^3 + x^4$. Find the intervals of increase or decrease of f , the local maximum and minimum values, and the intervals of concavity and inflection points. Use this information to sketch the graph of f .
9. Sketch the curve $y = x^3 - 3a^2x + 2a^3$, where a is a positive constant. What do the members of this family of curves have in common? How do they differ from each other?