Worksheet 3, Math 1A
Differentiation Rules

Monday, October 7, 2013

1. Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions that are sufficiently differentiable for the following questions to make sense, and let $F(x) = f(x)g(x)$.
   - Show that $F' = f''g + 2f'g' + fg''$.
   - Find similar formulas for F''' and $F^{(4)}$.
   - Guess a formula for $F^{(n)}$.
2. Use the chain rule and the product rule to prove the quotient rule.
3. Suppose that $y = f(x)$ is a curve that always lies above the x-axis and never has a horizontal tangent, where f is differentiable everywhere. For what value of y do we have that the rate of change of y^5 with respect to x is eighty times the rate of change of y with respect to x? Is it necessarily the case that there is such a point on this curve?
4. Write $|x| = \sqrt{x^2}$.
   - Use the chain rule to show that $d/dx\,|x| = x/\,|x|$.
   - If $f(x) = |\sin x|$, find $f'(x)$ and sketch the graphs of f and f'. Where is f not differentiable?
   - If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graphs of g and g'. Where is g not differentiable?
5. If $y = f(g(x))$, where f and g are twice differentiable functions, find y''.
6. Find the 999th derivative of $f(x) = xe^{-x}$. *Hint:* Don't directly compute 999 derivatives.
7. When does $f(x) = x + 2\sin x$ have a horizontal tangent? How about $g(x) = e^x \cos x$?
8. Prove that $d/dx(\csc x) = -\csc x \cot x$ using the derivatives of sin and cos.