Name:

Math 110, Section 103, Quiz 12
Wednesday, November 15, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ There is exactly one inner product on the vector space $\mathbb{R}^n$.

b. ________ An inner product may be used to define angles in a vector space.

c. ________ An inner product may only be defined on a finite-dimensional vector space.

Solution. F T F

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Exercise. Prove that the map $\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx$ is not an inner product on the space of *piecewise* continuous real-valued functions on the interval $[0,1]$.

Hint: For a counterexample to one of the axioms, what's an example of a piecewise continuous function that is almost, but not quite, the zero function?

Solution. Let $f : [0,1] \to \mathbb{R}$ be the piecewise continuous function defined by

$$f(x) = \begin{cases} 1, & x = 0 \\ 0, & x \in (0,1] \end{cases}$$

Then f is not the zero function, but $\langle f, f\rangle = \int_0^1 f(x)^2\,dx$ is zero because $f(x)^2 = 0$ for all points in the interval $(0,1)$. Note that eliminating the strict continuity requirement allows f to have a sudden jump, which can change a function (such as the zero function) at a single point without changing the corresponding integral.