Name:

Math 110, Section 103, Quiz 11
Wednesday, November 8, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ The Jordan canonical form of a generalized eigenspace is diagonal when its corresponding dot diagram is a single row.

b. ________ Two matrices in $M_{n\times n}(\mathbb{C})$ are similar iff their Jordan canonical forms are the same.

c. ________ If T is a linear operator whose characteristic polynomial splits with two distinct eigenvalues, then different pairs of dot diagrams corresponding to these eigenvalues may yield the same Jordan canonical form.

Solution. T T F

✷

Exercise. Suppose that A is a matrix whose characteristic polynomial is given by

$$\mathrm{char}_A(\lambda) = (-1)^6(\lambda-1)^2(\lambda-3)^4$$

and such that the geometric multiplicities of the eigenvalues are $m_g(1) = 2$ and $m_g(3) = 2$. What dot diagrams are possible for each of these two eigenvalues? Write out one Jordan canonical matrix J that could correspond with A. Which dot diagram or dot diagrams correspond with J?

Solution. The algebraic multiplicity of an eigenvalue is equal to the number of dots in its corresponding dot diagram, and the geometric multiplicity is equal to the number of columns. Thus for each eigenvalue we only have a few possibilities for dot diagrams:

$\lambda = 1$:

$$\begin{array}{cc} \bullet & \bullet \end{array}$$

$\lambda = 3$:

$$\begin{array}{cc} \bullet & \bullet \\ \bullet & \\ \bullet & \end{array} \qquad\qquad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array}$$

In particular, a Jordan canonical form for A which corresponds with the (only) dot diagram for $\lambda = 1$ and with the second dot diagram for $\lambda = 3$ is given by

$$J = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$