Name:

Math 110, Section 105, Quiz 10

Wednesday, November 1, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If $T : V \to V$ is a linear operator on a vector space V, then any generalized eigenspace K_λ is T-cyclic.

b. ________ A finite-dimensional vector space V with an *arbitrary* linear operator $T : V \to V$ splits into a direct sum of generalized eigenspaces.

c. ________ A generalized eigenspace may contain no eigenvectors.

Solution. F F F

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Exercise. Find a basis for the generalized eigenspace K_{-1} for the linear transformation $L_A : \mathbb{R}^4 \to \mathbb{R}^4$ given by left matrix multiplication by

$$A = \begin{pmatrix} -1 & 2 & 1 & -3 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Solution. To work with the generalized eigenspace K_{-1}, we find the stabilizing constant of the operator

$$U = A + I = \begin{pmatrix} 0 & 2 & 1 & -3 \\ 0 & 2 & 1 & -3 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Row reducing shows that $\dim(N(U^1)) = 2$. Multiplying, we can compute that

$$U^2 = \begin{pmatrix} 0 & 4 & 4 & -8 \\ 0 & 4 & 4 & -8 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Row reducing again, we conclude that $\dim(N(U^2)) = 2$ as well. This shows that the stabilizing constant of U is 1, and so $K_{-1} = N(U)$ is actually just the eigenspace E_{-1} of A. In particular, we find that $N(U)$ is specified by $x_2 = x_4$ and $x_3 = x_4$ with x_1, x_4 free. Thus a basis is given by $\beta = \{(1, 0, 0, 0), (0, 1, 1, 1)\}$.