Name:

Math 110, Section 103, Quiz 10
Wednesday, November 1, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ A Jordan block is diagonalizable iff it is 1×1.

b. ________ If $T : V \to V$ is a linear operator on an *arbitrary* vector space V, then the generalized eigenspace K_λ can be represented as $N((T - \lambda I)^k)$ for a fixed integer k.

c. ________ Every subspace of a T-invariant space is T-invariant.

Solution. T F F

✷

Exercise. Find a basis for the generalized eigenspace K_6 for the linear transformation $L_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by left matrix multiplication by

$$A = \begin{pmatrix} 7 & 1 & 2 \\ 1 & 5 & 0 \\ -1 & 1 & 6 \end{pmatrix}$$

Solution. To work with the generalized eigenspace K_6, we find the stabilizing constant of the operator

$$U = A - 6I = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{pmatrix}$$

Row reducing shows that $\dim(N(U^1)) = 1$. Multiplying, we can compute that

$$U^2 = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & -2 & -2 \end{pmatrix}$$

Again by row reducing, we see that $\dim(N(U^2)) = 2$. Multiplying once more, we find

$$U^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We find now that U^3 is the zero operator, so this implies that $\dim(N(U^3)) = 3$ can't increase any further. Thus the stabilizing constant of U is 3, and we have that $K_6 = N(U^3) = \mathbb{R}^3$. A fine choice of basis for K_6 is thus the standard basis on $\mathbb{R}^3$.