Name:

Math 110, Section 101, Quiz 10
Wednesday, November 1, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If $T : V \to V$ is a linear operator on a vector space V, then any generalized eigenspace K_λ is T-invariant.

b. ________ For eigenvalues $\lambda \neq \mu$ of a linear operator, the generalized eigenspaces K_λ and K_μ have intersection equal to the trivial subspace $\{0\}$.

c. ________ Any polynomial in $P(\mathbb{C})$ splits over $\mathbb{C}$.

Solution. T T T

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Exercise. Find a basis for the generalized eigenspace K_4 for the linear transformation $L_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by left matrix multiplication by

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -3 & 0 & -1 \\ 3 & 4 & 5 \end{pmatrix}$$

Solution. To work with the generalized eigenspace K_4, we want to find the stabilizing constant of the operator

$$U = A - 4I = \begin{pmatrix} -3 & -2 & 1 \\ -3 & -4 & -1 \\ 3 & 4 & 1 \end{pmatrix}$$

Row reducing shows that $\dim(N(U^1)) = 1$. Multiplying, we can compute that

$$U^2 = \begin{pmatrix} 18 & 18 & 0 \\ 18 & 18 & 0 \\ -18 & -18 & 0 \end{pmatrix} = 18 \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$$

Again by row reducing, we see that $\dim(N(U^2)) = 2$. Multiplying once more, we find

$$U^3 = \begin{pmatrix} -108 & -108 & 0 \\ -108 & -108 & 0 \\ 108 & 108 & 0 \end{pmatrix} = -108 \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$$

In particular, we see that $\dim(N(U^3)) = 2$ is the same as that for U^2, and so the stabilizing constant of U is 2. Thus $K_4 = N(U^2)$, and we see that a basis of K_4 is $\beta = \{(-1, 1, 0), (0, 0, 1)\}$.