Math 110, Section 103, Quiz 7
Wednesday, October 11, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. _____ If A is an $n \times n$ matrix with rank n, then the reduced row echelon form of A is I_n.

b. _____ If A is an invertible matrix, then $\det(A^{-1}) = -(\det(A))^{-1}$

c. _____ For any two $n \times n$ matrices A and B, $\det(AB) = \det(BA)$.

Solution. T F T

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Exercise. Evaluate $\det(A)$ for the following matrix A by using row reduction to transform the matrix to a simpler form.

$$A = \begin{pmatrix} 1 & 0 & -2 & -1 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix}$$

Solution. We use row reduction to reduce A to an upper triangular form, using only the operation of adding a multiple of one row to another so that the determinant is preserved at each step. We have

$$A \sim \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -5 & -1 \\ 0 & 4 & -1 & 1 \\ 0 & 3 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 19 & 5 \\ 0 & 0 & 19 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 19 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus $\det(A)$ is given by the product of the entries along the main diagonal of this upper triangular matrix, so $\det(A) = 19$.