Math 110, Section 101, Quiz 7

Wednesday, October 11, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If A is an $m \times n$ matrix with row reduced echelon form B, then the number of nonzero rows of B is the rank of A.

b. ________ Determinants of $n \times n$ matrices are preserved by all elementary row and column operations.

c. ________ If the rows of an $n \times n$ matrix A are linearly independent, then $\det(A) \neq 0$.

Solution. T F T

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Exercise. If A is the following invertible 4×4 matrix, compute $\det(A^{-1})$.

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & -2 & 0 & -4 \end{pmatrix}$$

Solution. Using the fact that $\det(A^{-1}) = 1/\det(A)$, we can just compute $\det(A)$. For this purpose, we will subtract a multiple of the first row from the second and fourth rows to reduce to an upper triangular form.

$$A \sim B := \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Then $\det(A) = \det(B)$, which is the product of the diagonal entries of B, or $1 \cdot (-2) \cdot 1 \cdot (-1) = 2$. Thus $\det(A^{-1}) = 1/2$.