Math 110, Section 103, Quiz 5
Wednesday, September 27, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If A is a nonzero $n \times n$ matrix, then for any $n \times n$ matrices B and C, if $AB = AC$, then $B = C$.

b. ________ If V is a vector space with finite bases α and β, then for any invertible linear transformation $T : V \to V$, we have $([T]_\beta^\alpha)^{-1} = [T^{-1}]_\alpha^\beta$.

c. ________ If A and B are two $n \times n$ matrices, then there is always a matrix Q such that $B = Q^{-1}AQ$.

Solution. F T F

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Exercise. If $\beta_1 = \{(2,5),(-1,-3)\}$ is a basis of $\mathbb{R}^2$ and $\beta_2 = \{e_1, e_2\}$ is the standard basis, then find a matrix Q such that $[T]_{\beta_2} = Q^{-1}[T]_{\beta_1}Q$ for any linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

Solution. The matrix Q should be the change of basis matrix from β_2 to β_1, which is $[\mathrm{Id}]_{\beta_2}^{\beta_1}$. Since β_2 is the standard basis, we know the value of the related matrix

$$[\mathrm{Id}]_{\beta_1}^{\beta_2} = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$$

which is formed by using the vectors in β_1 as the column vectors. In particular, we have that the desired matrix Q is given by

$$Q = [\mathrm{Id}]_{\beta_2}^{\beta_1} = ([\mathrm{Id}]_{\beta_1}^{\beta_2})^{-1} = \frac{1}{2 \cdot (-3) - (-1) \cdot 5} \begin{pmatrix} -3 & 1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$$