Math 110, Section 105, Quiz 4
Wednesday, September 20, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{n\times p}(\mathbb{R})$, then $AB \in M_{m\times p}(\mathbb{R})$.

b. ________ Matrix multiplication is commutative.

c. ________ Let L denote the map which takes an $m\times n$ matrix A and returns the corresponding linear transformation $L_A : F^n \to F^m$ obtained by left multiplication by A. Then L is a linear transformation.

Solution. T F T

✷

Exercise. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation which maps (x, y, z) to

$$T(x, y, z) = (x - y + z, x - z, y)$$

Use matrix multiplication to compute $T(T(x, y, z))$.

Solution. The matrix corresponding to T in terms of the standard basis for $\mathbb{R}^3$ is

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Multiplying this matrix by itself we get

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 1 & -2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

Multiplying by the vector (x, y, z) gives

$$T(T(x, y, z)) = \begin{pmatrix} 0 & 0 & 2 \\ 1 & -2 & 1 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2z \\ x - 2y + z \\ x - z \end{pmatrix}$$