Math 110, Section 103, Quiz 1
Wednesday, August 30, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ The cancellation law for vector addition (that for any vectors x, y, and z, if $x+z = y+z$, then $x = y$) is an axiom (assumed property) of vector spaces.

b. ________ The set of polynomials of a fixed degree n with real coefficients forms a vector space over $\mathbb{R}$ with the standard polynomial addition and scalar multiplication.

c. ________ A vector space may be defined using the integers as its set of scalars.

Solution. F, F, F.

✷

Exercise. Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{Q}\}$, and define an addition and scalar multiplication by

$$(a_1, a_2) + (b_1, b_2) := (a_1 + b_2, a_2 + b_1), \qquad c(a_1, a_2) = (ca_1, ca_2)$$

Prove that the set V equipped with the above addition and scalar multiplication is *not* a vector space over $\mathbb{Q}$.

Solution. There are a variety of reasons why this fails to give a vector space. One example is that the vector addition is not commutative:

$$(1, 2) + (1, 3) = (1 + 3, 2 + 1) = (4, 3)$$

while

$$(1, 3) + (1, 2) = (1 + 2, 3 + 1) = (3, 4)$$