Math 110, Section 101, Quiz 1
Wednesday, August 30, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. _____ It is an axiom (assumed property) of a vector space that $a(x+y) = ax + ay$ for any vectors x and y and any scalar a.

b. _____ The set of all polynomials with integer coefficients forms a vector space over the rational numbers $\mathbb{Q}$ with the standard polynomial addition and scalar multiplication.

c. _____ The sum of two vectors x and y in $\mathbb{R}^2$ can be thought of as the diagonal of the parallelogram having x and y as adjacent sides.

Solution. T, F, T.

✷

Exercise. Let $M_{2,3}(\mathbb{Q})$ be the vector space of 2×3 matrices with rational entries. Give two distinct, nonzero matrices A and B in $M_{2,3}(\mathbb{Q})$, and compute:

- Their sum $A + B$
- The additive inverse $-A$
- The product $(5/2)A$

Solution. A matrix with dimensions 2×3 has two rows and three columns, so two examples of (nonzero) such matrices are

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Then their sum is given by

$$A + B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

the additive inverse is given by

$$-A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the scalar product is given by

$$(5/2)A = \begin{bmatrix} 5/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$